

Marwari college Darbhanga

Subject---physics(Hons)

Class—B.Sc. part 1

Paper—2 ; Group—A

Topic—Thermal physics (Maxwell's distribution law in terms of Velocity)

Lecture series—09

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Maxwell's Distribution Law in Terms of Velocity

Root mean square velocity:-

The **root mean square velocity** is the **square root** of the **average** of the **square** of the **velocity**. As such, it has units of **velocity**. The reason we use the **rms velocity** instead of the **average** is that for a typical gas sample the net **velocity** is zero since the particles are moving in all directions.

$$E_k = \frac{3}{2} RT$$

$$E_k = \frac{1}{2} m v^2$$

setting these two equal and solving for the average square velocity we get

$$v^2 = \frac{3 RT}{m}$$

The root mean square velocity or v_{rms} is the square root of the average square velocity and is

$$v_{\text{rms}} = \sqrt{\frac{3 RT}{M}}$$

Where M is equal to the molar mass of the molecule in kg/mol. The root mean square velocity is the square root of the average of the square of the velocity. As such, it has units of velocity. The reason we use the rms velocity instead of the average is that for a typical gas sample the net velocity is zero since the particles are moving in all directions. This is a key formula as the velocity of the particles is what determines both the diffusion and effusion rates.

The Maxwell-Boltzmann distribution is used to determine how many molecules are moving between velocities v and $v + dv$. Assuming that the one-dimensional distributions are independent of one another, that the velocity in the y and z directions does not affect the x velocity, for example, the Maxwell-Boltzmann distribution is given by

$$\frac{dN}{N} = \left(\frac{m}{2\pi k_b T} \right)^{1/2} \exp \left[\frac{-mv^2}{2k_b T} \right] dv \quad (1.)$$

where

- dN/N is the fraction of molecules moving at velocity v to $v + dv$,
- m is the mass of the molecule,
- k_b is the Boltzmann constant, and
- T is the absolute temperature.¹

Additionally, the function can be written in terms of the scalar quantity speed v instead of the vector quantity velocity. This form of the function defines the distribution of the gas molecules moving at different speeds, between v_1 and v_2 , thus

$$f(v) = 4\pi v^2 \left(\frac{m}{2\pi k_b T} \right)^{3/2} \exp \left[\frac{-mv^2}{2k_b T} \right]$$

Finally, the Maxwell-Boltzmann distribution can be used to determine the distribution of the kinetic energy of for a set of molecules. The distribution of the kinetic energy is identical to the distribution of the speeds for a certain gas at any temperature.² The Maxwell-Boltzmann distribution is a probability distribution and just like any such distribution, can be characterized in a variety of ways including.

- **Average Speed:** The average speed is the sum of the speeds of all of the particles divided by the number of particles.
 - **Most Probable Speed:** The most probable speed is the speed associated with the highest point in the Maxwell distribution. Only a small fraction of particles might have this speed, but it is more likely than any other speed.
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